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THERMODYNAMICS OF CHARGED ADS BLACK HOLES IN RAINBOW GRAVITY

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ABSTRACT: The thermodynamic properties of charged Adds black holes are studied in this article in terms of rainbow gravity. The distorted temperature is determined by applying the Heisenberg Uncertainty Principle and the modified dispersion relation. Furthermore, we determine the charge's heat capacity and examine its thermal stability within the framework of rainbow gravity. Additionally, we deduce the pressure from the cosmological constant and study the critical dynamics of the liquid-gas system with an expanding phase space (P + U), where the black hole charge remains constant. A phase transition is also indicated by the unique swallow tail feature of the Gibbs function, which is revealed by a study of the function. In addition, we find that with a given mass of the test particle a black hole with a fixed charge and zero temperature can have an indefinitely rising heat capacity.

Keywords: *Thermodynamics, Charged black holes, Ads black holes, Rainbow gravity, Black hole thermodynamics, Statistical mechanics*

1.INTRODUCTION

Although the well-known Lorentz symmetry is one of the most fundamental symmetries in nature, some studies indicate that it might be broken at the UV limit. Any modification to the Lorentz symmetry will likewise alter the classical energy-momentum dispersion connection due to their interdependence. Computations of loop quantum gravity suggest that dispersion relations are tunable. Concurrently, double special relativity was constructed using the distorted energy-momentum dispersion connection. The greatest achievable speed of light is represented theoretically by the Planck energy, which is believed to be an extra constant for the maximum energy scale in nature. It offers a fresh viewpoint on the potential advantages of special relativity for microcosmic physics. Te theory is extended to curved spacetime by "gravity's rainbow," a concept put out by Joao Magueijo and Lee Smolin. The hypothesis states that viewers with varying energies will interpret spacetime geometry differently due to the fluctuating energy of the test particle. The geometry of spacetime will be defined by rainbow metrics, which are energy dependent, unlike the ideas of general gravity theory. The energy-momentum dispersion relation can be reformulated as follows thanks to the nonlinear Lorentz transformation:

$$E^2 f^2\left(\frac{E}{E_p}\right) - p^2 g^2\left(\frac{E}{E_p}\right) = m^2, \qquad (1)$$

in which case E_P Planck is the source of our energy.. The talents of the

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rainbow $f(E/E_P)$ and $g(E/E_P)$ must carry out their responsibilities

$$\lim_{E/E_P \to 0} f\left(\frac{E}{E_P}\right) = 1, \qquad \lim_{E/E_P \to 0} g\left(\frac{E}{E_P}\right) = 1. \tag{2}$$

In this case, the distorted energy-momentum dispersion relationship equation applies. (1) will revert to the classical one when the energy of the test particle is considerably diminished from E_P. The metric has changed as a result of this energy-dependent alteration to the dispersion relation. $h(E)$ The gravitational spectrum [9] potentially be adjusted.

The energy dependency of the frame fields is located.

$$e_0(E) = \frac{1}{f(E/E_P)}\tilde{e_0}, \qquad e_i(E) = \frac{1}{g(E/E_P)}\tilde{e_i}, \tag{4}$$

In this example, the energy-independent frame fields are shown by the tilde quantity. This results in a single-parameter Einstein equation.

$$G_{\mu\nu}\left(\frac{E}{E_P}\right) + \Lambda\left(\frac{E}{E_P}\right) g_{\mu\nu}\left(\frac{E}{E_P}\right) = 8\pi G\left(\frac{E}{E_P}\right) T_{\mu\nu}\left(\frac{E}{E_P}\right), \tag{5}$$

where $G_{\mu\nu}(E/E_P)$ and $T_{\mu\nu}(E/E_P)$ Are reliant on energy. Furthermore, alongside the Einstein tensor, there is the energy-momentum tensor. $\Lambda(E/E_P)\, G(E/E_P)$ The cosmological constant, which is energy dependent, and the Newton constant are both stated. Although various forms of rainbow functions have been documented in earlier study, the ones we will employ the most in our investigation are:

$$f\left(\frac{E}{E_P}\right) = 1, \qquad g\left(\frac{E}{E_P}\right) = \sqrt{1 - \eta\left(\frac{E}{E_P}\right)^n}, \tag{6}$$

This is extensively used in a variety of sources. Recent research has focused on studying Schwarzschild AdS black holes, Reissner-Nordstrom black holes, and Schwarzschild black holes within the context of rainbow gravity Ahmed Farag Alia, Mir Faizald, and Mohammed M. Khalile used the Heisenberg Uncertainty Principle (HUP) to analyze the changed temperature distribution surrounding charged AdS black holes in the setting of rainbow gravity. $(\mathrm{HUP}), \breve{E} = \Delta\acute{p} \sim 1/r_+.$ A charged AdS black hole's thermodynamic properties under rainbow gravity circumstances are examined in this study using the standard $(\mathrm{HUP}), \breve{E} = \Delta\acute{p} \sim 1/r_+.$ We look into how the mass of the test particle influences the thermodynamic parameters of charged AdS black holes. The following is how this document is structured. The distorted temperature is estimated in the following section using the HUP and the modified dispersion relation. A constant charge is also used to calculate thermodynamic stability and heat capacity. As explained in Section 3 of the article, charged AdS black holes behave like liquid-gas systems with the cosmological constant controlling the pressure. Instead of being a thermodynamic variable, the black hole charge Q is considered as a constant external number in this case. We also search for Gibbs free energy and the swallow tail's distinctive behavior. Section 4 is the final section, which includes the argument.

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2.THE THERMAL STABILITY

For modified charged AdS black holes, the line element can be written as [15] in terms of rainbow gravity.

$$ds^2 = -\frac{N}{f^2}dt^2 + \frac{1}{Ng^2}dr^2 + \frac{r^2}{g^2}d\Omega^2, \qquad (7)$$

Where

$$N = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{l^2}. \qquad (8)$$

In general, −3/l2 = Λ Having this as a constant is a given. The test particle's energy has no bearing on N since the rainbow functions f and g are where any energy dependence in the energy-independent coordinates would manifest. When compared to the standard temperature T0, the distorted temperature in the gravity rainbow was [15]

$$T = -\frac{1}{4\pi}\lim_{r\to r_+}\sqrt{\frac{-g^{11}}{g^{00}}}\frac{\left(g^{00}\right)'}{g^{00}} = \frac{g\left(E/E_P\right)}{f\left(E/E_P\right)}T_0, \qquad (9)$$

What is the address r_+ The radius of the horizon is the distance between an observer and the farthest point visible to them. Despite the fact that the metric is based on the energy of the test particle, the usual Heisenberg uncertainty principle (HUP) remains applicable in Gravity's Rainbow. For the sake of simplicity, we shall use n = 2 in the following explanation. We can get (1) and (6) by combining them.

$$g = \sqrt{1-\eta G_0 m^2}\sqrt{\frac{r_+^2}{r_+^2+\eta G_0}}, \qquad (10)$$

pertaining to the case $G_0 = 1/E_P^2$, m is the test particle's mass, and η is a constant parameter. In general, [22] provided the standard temperature.

$$T_0 = \frac{1}{4\pi}\left(\frac{1}{r_+} + \frac{3r_+}{l^2} - \frac{Q^2}{r_+^3}\right). \qquad (11)$$

Equations (6) and (10), when applied to charged AdS black holes in rainbow gravity, yield the temperature of these objects.

$$T = gT_0 = \frac{1}{4\pi k}\sqrt{\frac{r_+^2}{r_+^2+\eta G_0}}\left(\frac{1}{r_+} + \frac{3r_+}{l^2} - \frac{Q^2}{r_+^3}\right), \qquad (12)$$

\

in which $k = 1/\sqrt{1-\eta G_0 m^2}$. You can find it easily $T = T_0$ on what date. $\eta = 0$. The results of Equation (12) show that there are when $T = 0$, One resembles an enormous black hole, whereas the other is not. $m^2 = 1/\eta G$. The temperature of black holes that maintain a constant anti-de Sitter radius, charge, and mass, is solely determined by the mass of the test particle, as stated in the second response. Black hole temperatures exhibit a negative correlation with the mass of the measuring particles. $m^2 = 1/\eta G$, with immediate effect At 0 degrees, the temperature remains constant. The minimal radius of a black hole is commonly established by the rainbow of gravity, which exhibits a correlation with the radius of the black hole remnant as its temperature approaches zero Nevertheless, according to our investigation, it is possible for any black hole to sustain a temperature of zero as long as the test particle's mass approaches its nominal value. However, establishing the existence of zero temperature in the vicinity of black holes may prove difficult due to general conditions. Typically, thermal stability can be evaluated by employing heat capacity, a metric that is also applicable to black hole systems .In contrast, a condition of instability possesses a negative heat

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capacity, while one that is stable possesses a positive heat capacity. Subsequent dialogues will center on the stability of black holes, placing particular emphasis on their thermal capacity. When N equals zero, the mass of charged AdS black holes can be calculated as follows:

$$M = \frac{1}{2}\left(r_+ + \frac{Q^2}{r_+} + \frac{r_+^3}{l^2}\right). \quad (13)$$

In light of the first law $dM = TdS$ The deformed temperature can be used to compute the changed entropy .

$$S = \int \frac{dM}{T} = \pi k r_+ \sqrt{r_+^2 + \eta G_0} + \pi k \eta G_0 \ln\left(r_+ + \sqrt{r_+^2 + \eta G_0}\right). \quad (14)$$

Please note that the subsequent sequence is logarithmic in nature. $S \approx \pi r_+^2 + (1/2)\pi\eta G_0 \ln(4r_+^2)$, This is analogous to the quantum correction elucidated in references [26-31]. We can manage by employing. $A = 4\pi r_+^2$ we can get $S \approx A/4 + (1/2)\pi\eta G_0 \ln(A/\pi)$. The observed outcome aligns with the conventional concept of entropy. $S = A/4$ when $\eta = 0,$ This is a frequent happening. The heat capacity can be determined by considering a constant electric charge.

$$C_Q = T\frac{dS}{dT} = \left(\frac{\partial M/\partial r_+}{\partial T/\partial r_+}\right) = 2\pi k \frac{\left(-Q^2 l^2 r_+^2 + l^2 r_+^4 + 3r_+^6\right)\left(r_+^2 + \eta G_0\right)^{3/2}}{3r_+^7 + \left(6\eta G_0 - l^2\right) r_+^5 + 3Q^2 l^2 r_+^3 + 2\eta G_0 Q^2 l^2 r_+}, \quad (15)$$

Notably, logarithmic is the following leading order.

$$S \approx \pi r_+^2 + (1/2)\pi\eta G_0 \ln(4r_+^2),$$

Interestingly, the leading order that follows is logarithmic. [26–31]. With $A = 4\pi r_+^2$ We can acquire

$$S \approx A/4 + (1/2)\pi\eta G_0 \ln(A/\pi).$$

The observed result aligns with the expected entropy. $S = A/4$ when $\eta = 0,$ It is a frequent phenomenon. The heat capacity can be calculated using a fixed charge

$$C_Q = T\frac{dS}{dT} = \left(\frac{\partial M/\partial r_+}{\partial T/\partial r_+}\right) = 2\pi k \frac{\left(-Q^2 l^2 r_+^2 + l^2 r_+^4 + 3r_+^6\right)\left(r_+^2 + \eta G_0\right)^{3/2}}{3r_+^7 + \left(6\eta G_0 - l^2\right) r_+^5 + 3Q^2 l^2 r_+^3 + 2\eta G_0 Q^2 l^2 r_+}, \quad (15)$$

. that demonstrate C_Q reduced to its most fundamental form with $\eta = 0.$. When it comes to heat capacity, there is a clear difference. $m^2 = 1/\eta G.$. In most cases, the heat capacity decreases as the temperature decreases. The study we have done so far does, however, reveal an interesting twist. The good news is that this is all just an observational effect, and the outcome provides solid evidence for the scientific validity of rainbow gravity. The test particle's mass modifies the magnitudes of the attributes of temperature, entropy, and heat capacity, but in no way affects these properties themselves, as shown in multiple cases above. Figures 1, 2, and 3 illustrate that three cases corresponding to zero, one, and two sites where the heat capacity diverges have been identified using numerical methodologies. In Figure 1, there is no visible phase transition; the phase is continuous. $l < l_c.$ Figure 2 depicts two stable phases as well as one divergent point. $C_Q > 0$ with $l = l_c$ Phases 1 and 2 correspond to a small black hole (SBH) and a massive black hole (LBH), respectively. Figure 3 depicts two divergence sites and three phases. $l > l_c.$ Phase 1 is defined as a continuous

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transition from an unstable phase. $C_Q < 0$ to a stable condition $C_Q > 0$ The second phase is unadulterated. $C_Q < 0$; phase 3 is a moment of steadiness with. $C_Q > 0$. Clearly, phase 1 matches the SBH phase, while phase 3 matches the LBH phase. However, between phases 1 and 3, there is a distinct unstable phase 2. The system appears to have to transit through a medium unstable state to get from phase 3 to phase 1, which could be described by a strange quark gluon plasma with a negative heat capacity

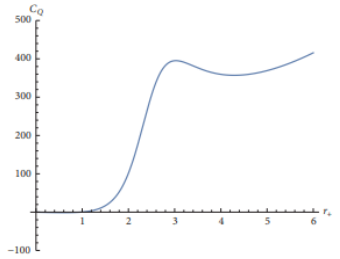


FIGURE 1: $C_Q - r_+$ diagram of charged AdS black holes in the rainbow gravity. It corresponds to $l = 6$. We have set $Q = 1, \eta = 1, m = 0$.

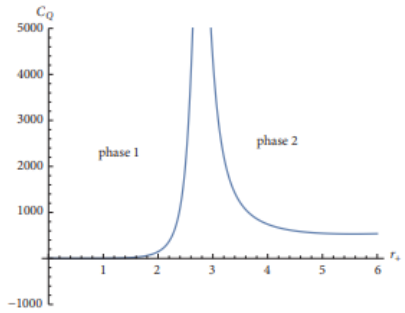


FIGURE 2: $C_Q - r_+$ diagram of charged AdS black holes in the rainbow gravity. It corresponds to $l = 7.05$. We have set $Q = 1, \eta = 1, m = 0$.

3. THE PHASE TRANSITION OF CHARGED ADS BLACK HOLES IN EXTENDING PHASE SPACE

Rainbow functions do, in fact, influence the $\Lambda(E/E_p)$ They do not influence the thermodynamic pressure when the cosmological constant is set Consequently, we arrive to the following connection:

$$G = H - TS = \frac{1}{2}\left(r_+ + \frac{Q^2}{r_+} + \frac{r_+^3}{l^2}\right) - kT(\pi r_+ \sqrt{r_+^2 + \eta G_0} + \pi\eta G_0 \ln\left(r_+ + \sqrt{r_+^2 + \eta G_0}\right), \quad (22)$$

In this work, we follow the work of David Kubiznak and Robert B. Mann to determine if the charged AdS black hole system in rainbow exhibits critical behavior similar to the liquid-gas system.

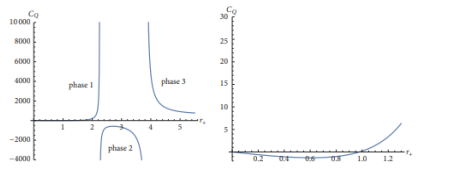


FIGURE 3: $C_Q - r_+$ diagram of charged AdS black holes in the rainbow gravity. It corresponds to $l = 8$, right one corresponds to an extending part of near $r_+ = 1$. We have set $\eta = 1, Q = 1, m = 0$.

The force of gravity remains constant. In extended phase space, we can obtain by applying (12) and

$$P = \frac{k}{2}\sqrt{\frac{r_+^2 + \eta G_0}{r_+^4}}T - \frac{1}{8\pi}\frac{1}{r_+^2} + \frac{1}{8\pi}\frac{Q^2}{r_+^4}. \quad (17)$$

Similarly, to [17], the crucial point is determined by

$$\frac{\partial P}{\partial r_+} = 0, \qquad \frac{\partial^2 P}{\partial r_+^2} = 0, \quad (18)$$

which leads to

$$r_c = \sqrt{\frac{2^{4/3}\eta G_0 Q^2 + 2^{4/3}Q^4 + 2Q^2(x+y)^{1/3} + 2^{2/3}(x+y)^{2/3}}{(x+y)^{1/3}}},$$
$$T_c = \frac{1}{2\pi k}\frac{r_c^2 - 2Q^2}{r_c^4 + 2\eta G_0 r_c^2}\sqrt{r_c^2 + \eta G_0}, \quad (19)$$
$$P_c = \frac{r_c^4 - 3Q^2 r_c^2 - 2\eta G_0 Q^2}{8\pi r_c^4 (r_c^2 + 2\eta G_0)},$$

where $x = \eta G_0 Q^2(\eta G_0 + Q^2)$ and $y = Q^2(\eta G_0 + Q^2)(\eta G_0 + 2Q^2)$. We can obtain

$$\frac{P_c r_c}{T_c} = k\frac{r_c^4 - 3Q^2 r_c^2 - 2\eta G_0 Q^2}{4r_c(r_c^2 - 2Q^2)\sqrt{r_c^2 + \eta G}}, \quad (20)$$

This illustrates that the crucial ratio is altered due to the effects of rainbow gravity. Significantly, (20) will return to its usual proportion with Typically, the pressure and temperature of charged AdS black holes necessitate positive real

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values. Since the year 19, when $P_c > 0$ and $T_c > 0$, we have

$$r_c^4 - 3Q^2 r_c^2 - 2\eta G_0 Q^2 > 0, \quad r_c > \sqrt{2}Q, \tag{21}$$

shows a distinction between Q and η. The $P - r_+$ The diagram is shown in Figure 4. As seen in Figure 4, charged AdS black holes under rainbow gravity exhibit a first-order phase transition similar to the Van-der-Waals system. $T < T_c$.

$$G = H - TS = \frac{1}{2}\left(r_+ + \frac{Q^2}{r_+} + \frac{r_+^3}{l^2}\right) - kT(\pi r_+ \sqrt{r_+^2 + \eta G_0} + \pi\eta G_0 \ln\left(r_+ + \sqrt{r_+^2 + \eta G_0}\right), \tag{22}$$

Figure 5 shows an example of this. The system has a first-order transition since the swallow tail behavior is portrayed in the figure of G.

4. CONCLUSION

Thermodynamic characteristics of charged AdS black holes subjected to rainbow gravity were studied in this study. By utilizing a modified dispersion relation in conjunction with HUP and a non-zero mass test particle, the distorted temperature in charged AdS black holes was computed. With a steady charge, we examined the changes in thermal capacity. Our results show that the phase structure is connected to the AdS radius l. At the time $l = l_c$

In the whole discussion of thermal capacity, there is exactly one disagreement. $l > l_c$ After locating two splits and three stages, we discovered two solid stages and one shaky one. It is brought to light that the charged AdS black holes operating under rainbow gravity are similar to the liquid-gas system. In addition, we proved $P - r_+$ Charged AdS black holes display basic characteristics within the rainbow gravity framework. The outcome is that rainbow gravity behaves like Van der Waals forces when η and Q are identical to equation (21). There is a distortion in the critical pressure forms due to the rainbow functions.

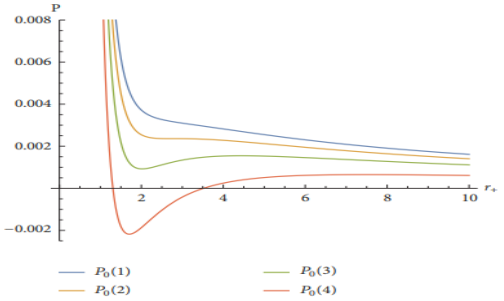


FIGURE 4: $P - r_+$ diagram of charged AdS black holes in the rainbow gravity. The temperature of isotherms decreases from top to bottom. The $P_0(1)$ line corresponds to one-phase for $T > T_c$. The critical state, $T_c = 0.0358$, is denoted by the $P_0(2)$ line. The lowest two lines correspond to the smaller temperature than the critical temperature. We have set $Q = 1, \eta = 1, m = 0$.

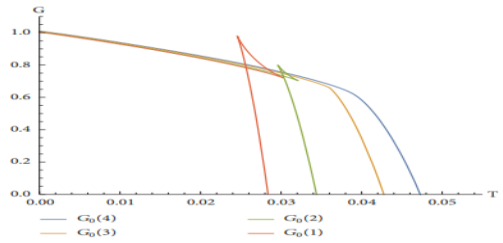


FIGURE 5: Gibbs free energy of charged AdS black holes in rainbow gravity. The blue line $G_0(3)$ corresponds to the critical pressure $P_c = 0.0024$, the line $G_0(4)$ corresponds to pressure $P > P_c$, and the others corresponds to pressure $P < P_c$. We have set $Q = 1, \eta = 1, m = 0$.

in addition to radius and temperature. At last, we resolved the first-order phase shift by analyzing the Gibbs free energy and discovering the usual "swallow tail" behavior. We found that the heat capacity, entropy, and temperature forms were unaffected by the test particle's mass. $m^2 = 1/\eta G$,

They have different amplitudes. Additionally, for charged AdS black holes with rainbow gravity, there is a particular mass of the test particle that would lead to divergent heat capacity and zero

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temperature. Availability of Data No evidence was found to back up this study. Possible Interest Conflicts Claiming to have no financial stake in the study's publication, the writers disclaim any bias in their work. Notes of Thanks The funding for this research was provided by the National Natural Science Foundation of China (Grant No. 11571342). We are quite grateful to them.

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